SOLVING TRANSIENT NONLINEAR HEAT CONDUCTION PROBLEMS BY PROPER ORTHOGONAL DECOMPOSITION AND FEM

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ABSTRACT A method of reducing the number of degrees of freedom and the overall computing times, so called Proper Orthogonal Decomposition (POD) combined with Finite Element Method (FEM) has been devised. The technique POD-FEM can be applied both to linear and nonlinear problems. At the first stage of the method standard FEM time stepping procedure is invoked. The temperature fields obtained for the first few time steps undergo statistical analysis yielding an optimal set of globally defined trial and weighting functions for the Galerkin solution of the problem at hand. The resulting set of Ordinary Differential Equations (ODEs) is of greatly reduced dimensionality when compared with the original FEM formulation. For linear problems, the set can be solved either analytically, resorting to the modal analysis technique, or by time stepping. In the case of nonlinear problems, only time stepping can be applied. The stress in this paper is on time stepping approach where the generation of the FEM-POD matrices, requiring some additional matrix manipulations, can be embedded in the assembly of standard FEM matrices. The gain in execution times comes from the significantly shorter time of solution of the set of algebraic equations at each time step. Included numerical results concern both linear and nonlinear problems. In the case of linear problems, the derived time stepping technique is compared with the standard FEM and the modal analysis. For nonlinear problems the proposed POD-FEM approach is compared with standard FEM. Good accuracy of the POD-FEM solver has been observed. Controlling the error introduced by the reduction of the degrees of freedom in POD is also discussed.